

Obscure waves in planetary atmospheres

Erdal Yiğit, and Alexander S. Medvedev

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The image shows a Lake Shore Cryotronics M91 FastHall Controller. It is a rectangular, silver-colored device with a large color touchscreen on the front. The screen displays four measurement panels: 'Continuity' (Next test), 'Contact Check' (2019-01-01 at 01:29, 1001 ms), 'Resistivity' (2019-01-01 at 01:28, 1000 ms), and 'FastHall™' (with a circular icon). The device has a 'Lake Shore CRYOTRONICS' logo on the top left and 'M91 FastHall' on the bottom right. A 'Measure Ready' indicator is visible on the front panel.

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Erdal Yiğit is an associate professor of physics at George Mason University in Fairfax, Virginia. **Alexander S. Medvedev** is a researcher at the Max Planck Institute for Solar System Research in Göttingen, Germany.



Obscure waves in planetary atmospheres

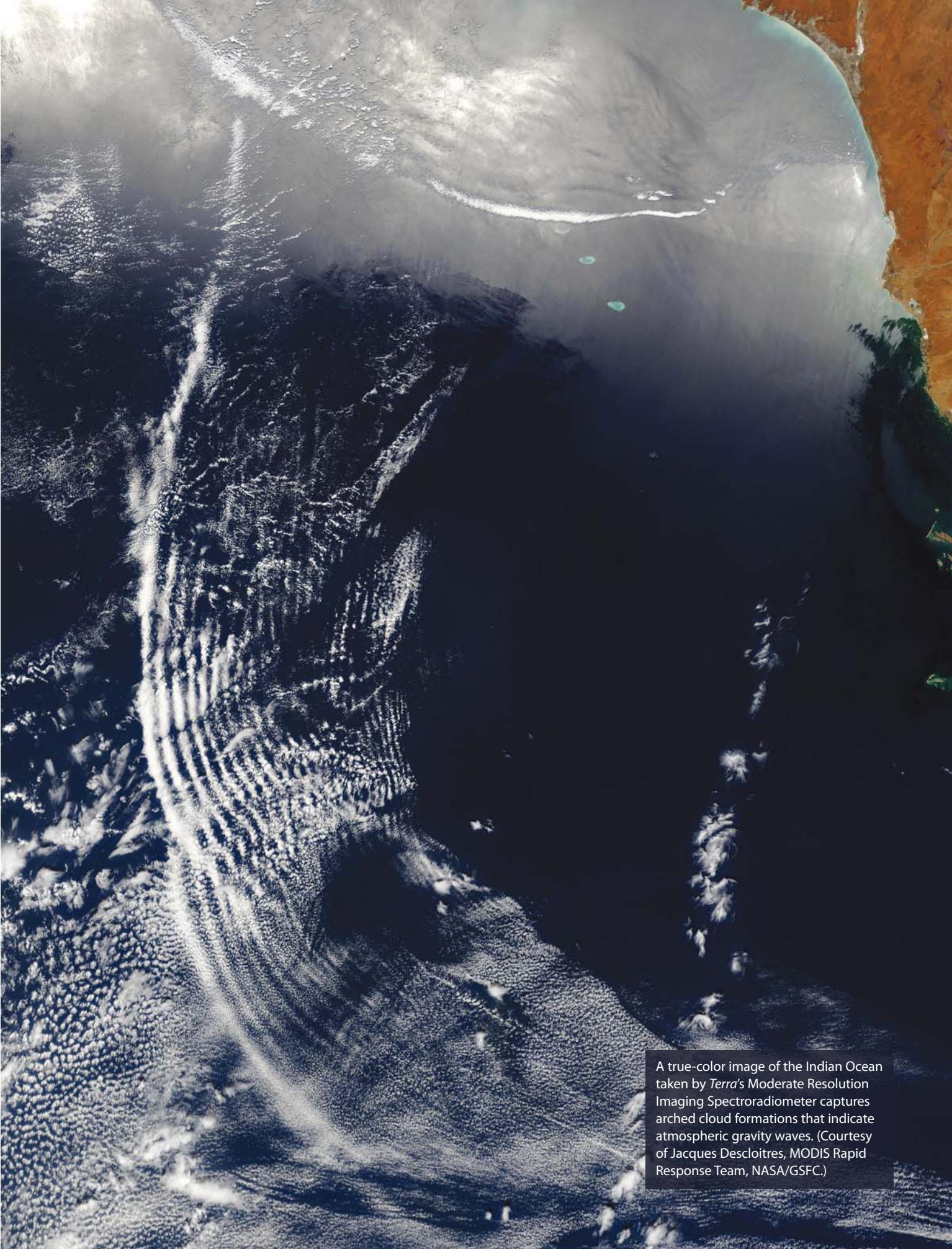
Erdal Yiğit and
Alexander S. Medvedev

**On Earth and on other planets,
internal gravity waves shape the
dynamics and thermodynamics
of the atmosphere.**

In 1893 the Arctic explorer Fridtjof Nansen experienced the phenomenon of dead water when some mysterious force brought his ship to an almost complete stop. Seafarers had long known that effect and superstitiously linked it to cursed drowned sailors holding onto the vessel. Nansen later asked a doctoral student, Vagn Walfrid Ekman, to explain the phenomenon scientifically. Ekman's thesis became the first academic work dedicated to explaining internal gravity waves. He attributed the dead-water phenomenon to lighter fresh water from melting sea ice sitting on top of saltier, denser seawater. The slowly moving vessel induced waves at the interface between the two layers beneath the sea surface, so its propulsion energy was wasted in making those waves instead of moving the ship. (For more on internal waves in the ocean, see the article by Callum Shakespeare on page 34 of this issue.)

Waves are ubiquitous in both the atmosphere and the ocean. The most familiar are sea waves: Almost everyone has observed how water waves travel along the ocean's surface, grow in size, and then break as they approach the coast. Waves can be viewed as a collective phenomenon resulting from oscillations around equilibrium of many neighboring parcels of a material. They can transfer energy without any net

transport of material. Gravity waves, like those considered by Ekman at the interface between two types of water, are similar to those on the sea surface that divide dense water and tenuous air. When a fluid parcel is displaced vertically, the forces that counteract the displacement are buoyancy and gravity. Consequently, such waves are alternatively called buoyancy or gravity waves. (The latter should not be confused



A true-color image of the Indian Ocean taken by *Terra's* Moderate Resolution Imaging Spectroradiometer captures arched cloud formations that indicate atmospheric gravity waves. (Courtesy of Jacques Desclotres, MODIS Rapid Response Team, NASA/GSFC.)

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with gravitational waves from the theory of general relativity as studied in cosmology.)

In a fluid with continuously varying density, oscillating parcels successively push the fluid above and below them during each cycle, so waves can propagate vertically in addition to cruising horizontally along the surface of constant density. To emphasize the distinction between surface and internal waves, those that can propagate both vertically and horizontally are called internal waves.

The atmosphere is a fluid with continuously varying density, so it is subject to the gravity waves described above. Although they usually get very little attention, they are omnipresent and important. Figure 1, a snapshot from a computer simulation conducted with a general circulation model, shows the signatures of gravity waves in the atmosphere at an altitude of around 250 km.

Gravity waves in the atmosphere

Air parcels rarely oscillate purely in the vertical direction. They have horizontal displacements as well, so their orbits are tilted ellipses. The more inclined the ellipse is, the longer it takes for a parcel to return to its initial state. Thus the periods of gravity waves are tightly linked to their horizontal and vertical wavelengths,¹ which range from a few minutes to many hours but are less than a day. The horizontal length scales of gravity waves extend from a few tens of kilometers to many hundreds but remain significantly smaller than the planetary radius.

Gravity waves represent only a part of the spectrum of vertically propagating internal waves. Planetary waves that define weather systems span thousands of kilometers and last for days, which makes gravity waves appear to be small-scale and have short periods. On the other hand, gravity waves are very large and low frequency when compared with sound waves, a familiar class of atmospheric oscillations.²

Earth's atmosphere can be divided into distinct layers based on its temperature profile, as depicted in figure 2. Internal gravity waves are continuously produced by various processes in all layers, but they primarily originate in the lower atmosphere, or troposphere. Since vertical displacements of air are the necessary condition for their excitation, any process that shifts parcels vertically can potentially generate an internal wave.³ Some obvious although infrequent sources of such waves in the atmosphere are tsunamis, hurricanes, earthquakes, and volcano eruptions. Air flows over hills and mountains are a more common generation mechanism. However, the main source of gravity waves in the troposphere is weather. Air parcels are forced to move vertically during such meteorological processes as convection, atmospheric fronts, cyclonic activity, and instability in wind systems. Because they have so many sources, gravity waves have phase velocities ranging from zero to an upper limit approaching the speed of sound in the atmosphere.

Gravity waves carry both energy and momentum. The en-

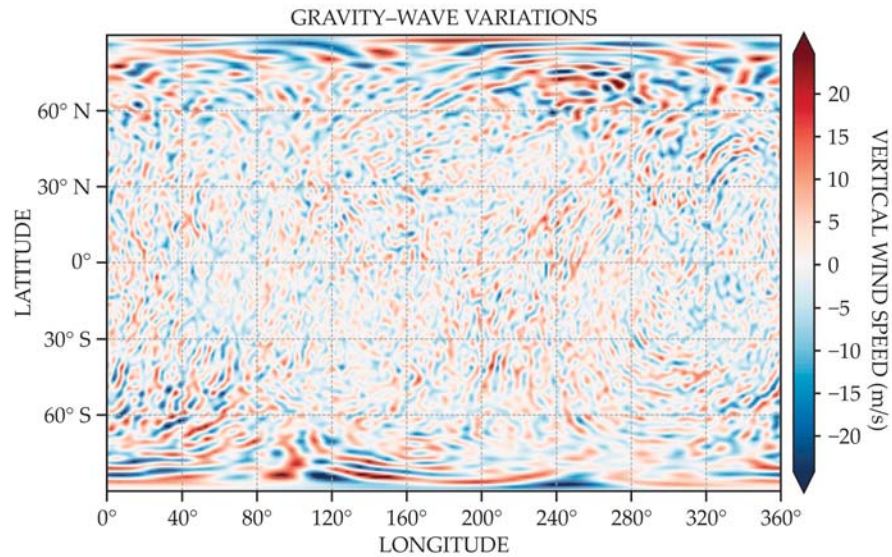


FIGURE 1. THE VERTICAL COMPONENT OF THE NEUTRAL WIND at an altitude of 250 km, simulated with the ground-to-topside model of atmosphere and ionosphere for aeronomy, is a proxy for gravity waves.¹⁶ This snapshot shows the latitude–longitude distribution of vertical wind speed on 3 January 2009, 00 UT. A horizontal resolution of $1^\circ \times 1^\circ$ allows the simulation to reproduce harmonics with horizontal wavelengths of 375 km or greater. (Courtesy of Yoshizumi Miyoshi.)

ergy of an individual wave or wavepacket is proportional to the product of its amplitude squared and the average density of the ambient air. As the wave propagates upward, its amplitude grows exponentially to compensate for the atmosphere's steeply declining density. That process changes the picture of motion in the middle and upper atmospheres: Gravity waves can't be ignored because they actually dominate. The middle and upper atmospheres are like a stormy ocean rippled with huge velocity disturbances accompanied by temperature fluctuations of tens of degrees, which makes the occasionally disastrous weather on Earth's surface seem like a relatively calm seafloor.

Historical retrospective

Atmospheric gravity waves were largely ignored until the golden age of aviation in the 1920s and 1930s, when understanding them became of practical interest. Aircraft occasionally encounter bumpiness when flying over hills and mountains, and often that bumpiness is related to gravity waves generated by that topography or to turbulence from breaking waves. Sometimes the bumpiness is accompanied by visual cues in the form of parallel bands of clouds left behind by propagating wavepackets. However, the most dangerous situation is clear-air turbulence (CAT), which, as the name suggests, is not manifested by clouds or linked to terrain features; its relation to weather phenomena is unclear. Understanding and predicting CAT is therefore important so that pilots can be prepared to deal with it. The dominant idea today is that CAT is the pseudoturbulence created by a superposition of many gravity waves,⁴ but forecasting it remains a challenge for scientists and the aviation industry.

Meteorologists developed a theoretical description for grav-

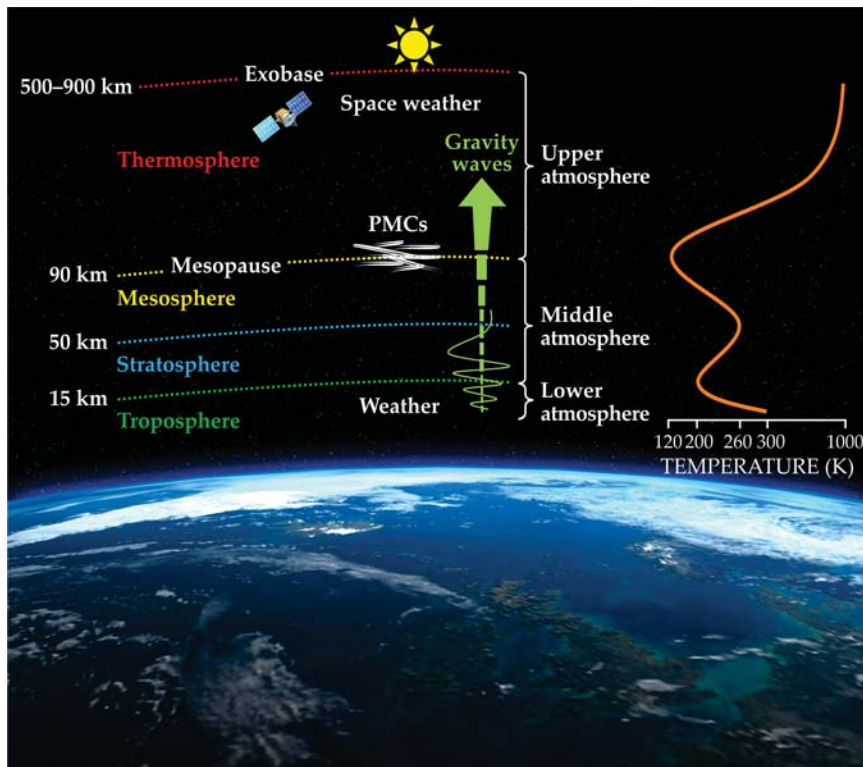


FIGURE 2. THE VERTICAL STRUCTURE of Earth's neutral atmosphere comprises layers that are identified by their temperature (orange line). Note that the temperature scale shown is nonlinear. The atmosphere's extent can be from 500 km to 900 km, depending on external forcing such as solar and geomagnetic activity. Weather processes and the generation of most energetic gravity waves take place in the troposphere, which is the lowest layer and extends about 10–15 km above Earth's surface. The middle atmosphere, containing the stratosphere and mesosphere, is above the troposphere and is separated from the thermosphere by the mesopause at an altitude of about 90 km. Polar mesospheric clouds (PMCs) that trace propagating gravity waves occur in the mesosphere. The upper atmosphere generally refers to the tenuous and hot thermosphere. Space weather processes—the effects of the Sun on the geospace environment—primarily influence the upper atmosphere.

ity waves in the early part of the 20th century⁵ but then lost interest in them, mainly because they were seen as weak and unimportant for weather. The energy associated with gravity-wave disturbances is, on average, only a small percentage of what is found in larger, smoother motions in the troposphere. Moreover, the waves were seen as a nuisance in the early days of numerical weather predictions because including them in simulations was computationally expensive, especially considering the computing power available.

Mathematically, gravity waves are solutions of the Navier–Stokes equation, the fundamental differential equation of fluid dynamics that forms the basis of all general circulation and numerical weather-prediction models. If gravity waves are omitted, time steps in numerical integration can be increased, and computer models run many times faster. A lot of ingenuity was applied to deriving and justifying hydrodynamic equations that precluded gravity-wave generation. Later observers recognized that ignoring such waves altogether worsens forecasts, especially during rapid weather transitions.

By the end of the 1960s, forecasters had reverted to using the full hydrodynamic equations, and all modern numerical weather models allow for gravity-wave generation. However, the idea of a simplified model that captures the useful properties of gravity waves without overloading computers still looks appealing.⁶

The beginning of the space age in the early 1950s also brought attention to gravity waves. Those were days of the Cold War, and both the Western and Eastern blocs were trying to build early warning systems for airborne attacks that relied on radio signals reflected by the ionosphere. Using rockets, scientists discovered that the atmosphere extends much higher than was previously thought. The first rockets were outfitted with aerosol ejectors that released columns of smoke in the

upper atmosphere. The smoke was expected to expand by diffusion and to be displaced by blowing winds. Instead, the scientists saw strange and intricate distortions reminiscent of long-lived meteor contrails.

It soon became clear that the upper atmosphere is not smooth, but is continuously perturbed by irregular disturbances of various temporal and spatial scales, such as gravity waves, turbulence, and solar radiation and flares. Other inputs—energetic particles, the geomagnetic field, and electromagnetic forces—also cause distortions in the upper atmosphere. Interestingly, in the mid 20th century, aeronomists—scientists who study the upper atmosphere—were largely unaware of gravity waves, and meteorologists had written them off as insignificant noise. At the end of the 1950s, Colin Hines, an aeronomist, was among the first to link numerous observed atmospheric features to gravity waves propagating from below. He was struck by the coupling and later described it in everyday terms: “The ionospheric regions would be like a light-weight tail wagged by a very massive dog, and they must respond to almost any disturbance created below.”⁷

The last two decades of the 20th century saw the proliferation of then revolutionary ground-based remote sensing techniques, including incoherent scatter radars based on reflection from density fluctuations, meteor radars exploiting ionized meteor trails, and pulsed lasers called lidars. Unlike rockets, those techniques do not disturb the atmosphere during observations and can almost continuously survey different altitudes up to about 100 km by analyzing reflected signals. The new measurements revealed many details about the structure and dynamics of the middle atmosphere and helped highlight the importance of gravity waves to atmospheric flows.

Perhaps the most often reported large-scale effect produced by smaller-scale waves is the creation of the coldest spot on

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Earth, which is over the summer pole near the mesopause (see the article by Bodil Karlsson and Ted Shepherd, *PHYSICS TODAY*, June 2018, page 30). Gravity waves propagating from below grow in amplitude and break in the middle atmosphere, just like water waves break in a surf zone. The deposited wave momentum drives pole-to-pole air flow in the middle atmosphere. The flow is directed from the summer into the winter hemisphere, similar to the current along the shore produced by breaking surface waves. In addition to pole-to-pole flow, upward and downward currents also move over the summer and winter poles, respectively, and complete a circulation cell. The cooling produced by the adiabatic expansion of rising air over the summer pole exceeds the heating from an almost constantly illuminating Sun; thus that location becomes the coldest spot on Earth. On a summer evening at dusk, one can look to the north and see the high-altitude polar mesospheric clouds that occasionally form in the cold far reaches. They are often crisscrossed with traces left by gravity waves.⁸

Gravity-wave physics

Waves generated in the lower atmosphere by various sources have no preferred horizontal direction. The net horizontal momentum transported up and absorbed in the middle and upper atmospheres would thus be zero. So how can such an apparently chaotic field of waves create a well-defined pole-to-pole current on breaking? As waves travel vertically, they pass through circumpolar winds. If the horizontal phase velocity of a wave matches the persistent local wind, the distinction between the wave and flow disappears. The wave is then absorbed by the flow and can no longer propagate. Since the wind jets are aligned east–west in the summer hemisphere and opposite in the winter hemisphere, they selectively filter out some incident gravity waves.⁹

Many of the surviving waves that enter the middle atmosphere eventually break or cease their growth due to dissipation. Breakup is a violent process that begins with the instability of large-amplitude waves and is followed by a transition to turbulent motion. Therefore, the onset of breakup strongly depends on wave amplitude. Dissipation acts gradually, and over time the energy and momentum carried by waves are returned to the local flow. The behavior of the middle atmosphere is so influenced by breaking gravity waves that it cannot be explained without them.

The notion of the surf zone led some aeronomists to believe that all gravity waves break down in the middle atmosphere. That impression was indirectly supported by a surge of numerical global climate models whose upper bounds were exactly where the middle atmosphere ends, about 90 km above Earth's surface. In fact, the mesosphere is more like a reef surf zone where many waves break but larger-scale waves roll over and continue cruising toward the beach. However, the analogy with oceans ends there because the upper atmosphere beyond the mesosphere is viscous.

The molecular viscosity of a gas is proportional to the mean free path of the constituent molecules; it is therefore inversely proportional to the density and grows exponentially with height.

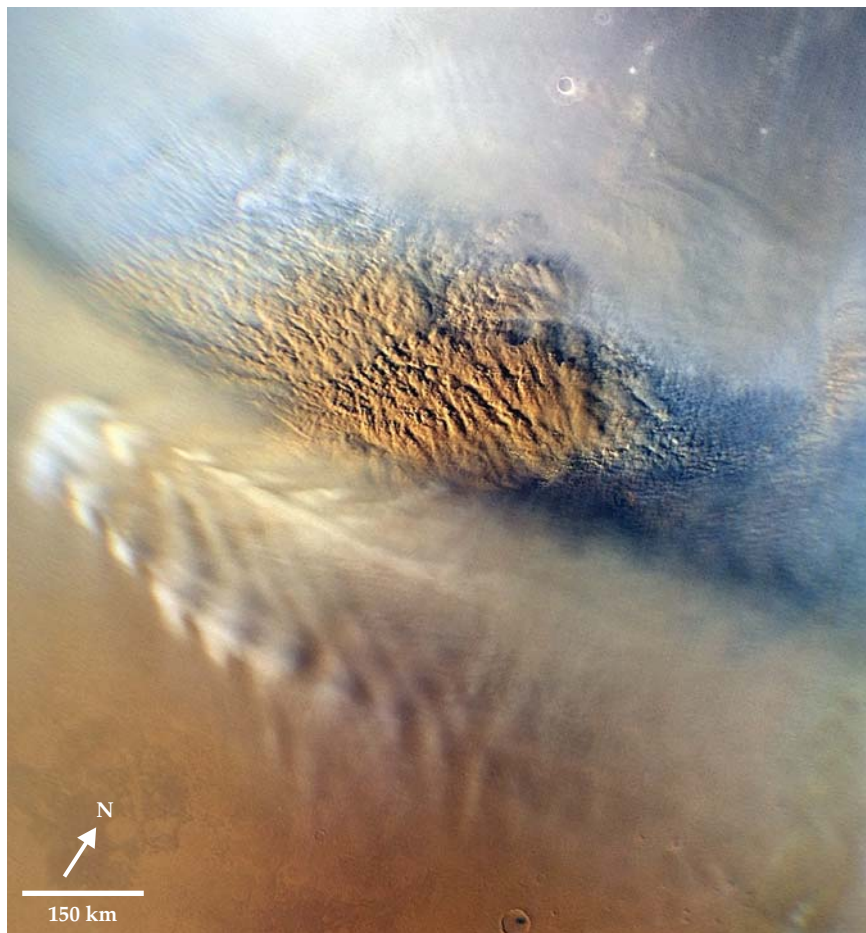


FIGURE 3. A MARTIAN DUST STORM was observed by the Mars Color Imager instrument on NASA's *Mars Reconnaissance Orbiter* on 7 November 2007 at around 3:00pm local time. This picture is centered on Utopia Planitia (53.6° N, 147.9° E); the seasonal polar north cap is seen at the top of the figure. Storms have variable durations; the dust storm pictured lasted for a day. Gravity-wave signatures are imprinted in the water ice clouds around Mie Crater in the bottom right of the image. The clouds form because of changes in atmospheric pressure and temperature from vertical air displacements caused by gravity waves that are excited by wind flow over a mountain. (Image courtesy of NASA/JPL-Caltech/MSSS. For more information, contact Malin Space Science Systems at www.msss.com.)

Competition between amplitude growth and exponentially increasing damping caused by molecular diffusion prevents gravity waves from breaking prematurely and helps some waves to propagate upward for several hundred kilometers and penetrate into what can already be considered space. The atmosphere as a continuous medium ends where collisions of molecules become so rare that their mean free paths greatly exceed spatial scales of motions. Thus the rarefied air at the upper edge of the atmosphere is already space for smaller-scale disturbances, but it can still sustain hydrodynamic waves with horizontal wavelengths of hundreds of kilometers.

The momentum transferred from breaking or dissipating waves to the local flow can accelerate or decelerate parcels of air in the upper atmosphere by hundreds of meters per second per day. That is a huge acceleration: Given that typical atmo-

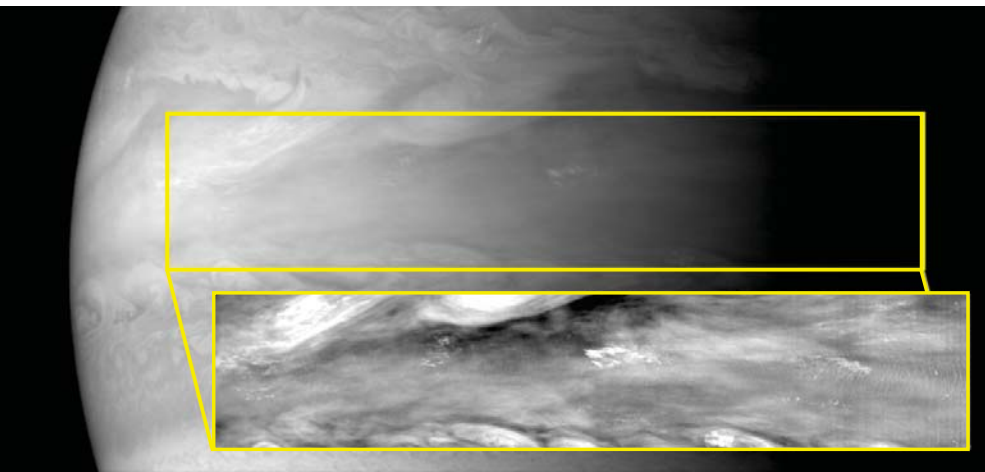


FIGURE 4. GRAVITY WAVES in Jupiter's equatorial atmosphere were detected in 2010 by the Multispectral Visible Imaging Camera aboard the *New Horizons* spacecraft. Jupiter is a gas giant and has no solid surface. Gravity waves are often generated by flow over mountains in terrestrial planets, but on Jupiter they are thought to be excited by convection deep below the visible clouds. Using the images from *New Horizons*, scientists found that the waves are moving much faster than the surrounding clouds. (Courtesy of NASA/Johns Hopkins University Applied Physics Laboratory/Southwest Research Institute.)

spheric wind velocities at those heights vary from several tens of meters per second to several hundreds, gravity-wave drag could kill them in about a day if no other forces were involved.

Gravity waves also affect the thermal state of the upper atmosphere. If a parcel of air is displaced up into a region with lower density, its temperature drops according to the first law of thermodynamics. Heating would then accompany its downward motion. In conservatively propagating waves, alternating phases of expansion and compression cancel each other out and produce no net input or loss of heat energy. On the other hand, when a wave attenuates with height, the expansions and compressions no longer cancel, and there is a net downward flux of sensible heat.¹⁰ Thus, gravity waves and molecular thermal conduction work together to refrigerate and maintain the stability of the upper thermosphere.

The remarkable progress with satellite and ground-based observing systems, numerical models, and increased computational capabilities over the past decade has helped to uncover the importance of gravity waves in the middle and upper atmospheres. They constitute a major mechanism of vertical coupling by redistributing momentum and energy between atmospheric layers and driving many dynamical phenomena.¹¹

Extraterrestrial atmospheres

Gravity waves are not unique to Earth; they can exist in all stably stratified fluids. Our solar system presents a great diversity of planets—including terrestrials, gas and ice giants, and dwarfs—and of atmospheres. Mars is the planet most visited by rovers and orbiters and the best known after Earth. Its thin atmosphere is reminiscent of Earth's mesosphere, but its rougher surface and stronger winds make its weather more volatile.

A thin, windy atmosphere and rough surface are also the ingredients for intensive gravity-wave generation. Remote sensing by orbiters confirmed that the gravity-wave activity on Mars is several times larger than on Earth, both in the lower and upper atmospheres. Figure 3 shows gravity-wave-induced variations in water-ice clouds as observed by NASA's *Mars Reconnaissance Orbiter* in 2007 during a local dust storm. Results from NASA's *Mars Atmosphere and Volatile Evolution* spacecraft suggest that the spectacular variability of the Martian thermosphere is produced by gravity waves traveling from sources near the planet's surface.¹²

Earth's closest planetary neighbor, Venus, has a scorching

hot atmosphere due to its enhanced greenhouse effect. Thick clouds that cover the entire planet hide many details in the Venusian troposphere. Nevertheless, observations show clear wave signatures imprinted on the cloud top, and the Japanese *Akasuki* orbiter recently detected very large scale gravity waves.¹³

Venus's atmosphere exhibits superrotation, a specific behavior in which the entire atmosphere rotates faster than the planet. Obviously, forces in the atmosphere must maintain that rotation, but their origin is unknown. Sophisticated numerical modeling provides ample indication that the mysterious force responsible for the superrotation is the torque delivered by gravity waves that originated in the atmosphere's hottest near-surface layers. As on Earth, they grow in amplitude with height and transfer their momentum to the mean flow on breaking at and above the cloud top.

Gravity waves have also been observed in the atmospheres of outer planets. In 1997 the *Galileo* probe, as it descended through the thermosphere and stratosphere of Jupiter, measured a temperature profile that revealed a gravity wavepacket propagating upward.¹⁴ Recently, NASA's *Juno* spacecraft captured images of wave trains—more or less equally spaced crests and troughs—that exhibit gravity-wave behavior. (An image of Jupiter's atmosphere taken by *Juno* is shown on this issue's cover.) The upcoming *Jupiter Icy Moons Explorer* mission, which is scheduled for launch in 2022 by the European Space Agency, is expected to survey the Jovian system for more than three years. One of its prime objectives is to investigate the structure, dynamics, and composition of the Jovian atmosphere and thus provide more insight into gravity wave dynamics.

The *New Horizons* probe passed by Pluto in 2015 and discovered optically thin haze layers that extend to altitudes greater than 200 km. The most plausible mechanism for maintaining those clouds is related to gravity waves generated by diurnal sublimation and flow over topography.¹⁵ On its way, the spacecraft also surveyed the Jovian system and captured an unprecedented view of gravity waves in Jupiter's equatorial atmosphere (see figure 4).

Living with gravity waves

Understanding the upper atmospheres of our own and other planets is necessary for learning how to navigate them. In the lower atmosphere, CAT challenges pilots and is thought to be produced by a superposition of many gravity waves. At higher

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altitudes, gravity waves can affect returning spacecraft, disturb satellite tracking, and skew GPS signals. Along with other larger-scale waves, they not only perturb atmospheric density at higher altitudes but also control the mean atmospheric state.

Space weather—the conditions surrounding Earth and other planets—is also remarkably coupled to the weather in the lower atmosphere by gravity waves. Space weather models are essential tools used to predict the Sun's effect on Earth. Geomagnetic storms are a fascinating manifestation of space weather caused by the interaction of solar energy and cosmic rays with Earth's magnetic field. During geomagnetic storms, the thermosphere-ionosphere system undergoes substantial changes, which can greatly affect propagation conditions of gravity waves. Thus, if we are to improve the predictive capabilities of space weather models, the influence of the whole spectrum of waves from below must be better understood and quantified.

On Mars and Venus, spacecraft perform aerobraking to modify their orbits by dipping into denser atmospheric layers of the thermosphere and mesosphere. Onboard accelerometers show that during every such maneuver, a spacecraft goes through a washboard-like profile of air density fluctuations. Understanding such gravity-wave-induced variability is important for planning and executing aerobraking operations, and for managing orbiters and ensuring the safety of their onboard instruments.

Continuing exploration and possible colonization of Mars will greatly increase the number of spacecraft that traverse the planet's atmosphere. As microsatellite architectures become

more popular, Mars may be explored and monitored by swarms of individual satellites. Density variations associated with gravity waves can greatly affect the motions of small satellite swarms and larger spacecraft alike.

Although they may seem insignificant and obscure in the daily lives of humans, internal gravity waves are fundamental in atmospheric dynamics because they link vertical layers. Future observations of other planetary atmospheres and even stars will help to uncover how universal the phenomenon really is.

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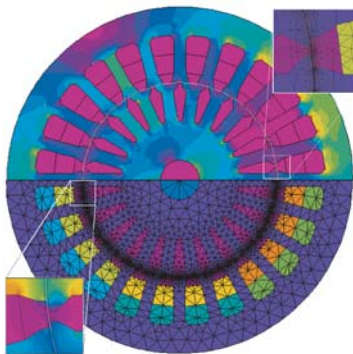
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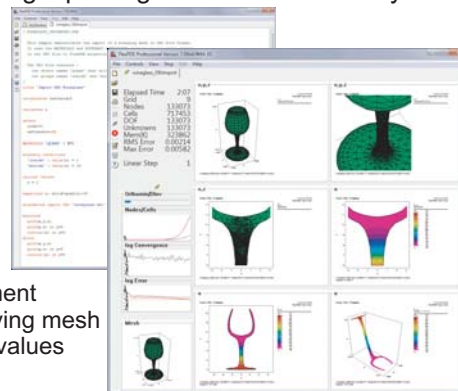
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